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Design of a Staggered-PRF MTI Filter

JAMES K. HSIAO

Search Radar Branch
Radar Division

and

FRANK F. KRETSCHMER, JR.

Target Characteristics Branch
Radar Division

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NAVAL RESEARCH LABORATORY
Washington, D.C.

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ABSTRACT

To avoid blind speed phenomena, staggered PRF MTI systems are used in modern radars for detection of high-speed targets. In this report a procedure is presented which considers both the effect of filter weights and the interpulse durations simultaneously. The design goal of this procedure is to yield a filter with minimum variations in the passband and maximum attenuation in the stopband.

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DESIGN OF A STAGGERED-PRF MTI FILTER

INTRODUCTION

An MTI (moving target indicator) filter commonly known as a comb filter exhibits a periodic property. Its frequency response folds over at the radar pulse repetition frequency (PRF). Thus a target having a doppler frequency which is an integer multiple of the radar PRF will be treated as stationary clutter and will be rejected by the MTI filter. This blind speed is a function of the radar operating frequency and the PRF which in turn determines the unambiguous range. For a modern radar which operates in the microwave range (L-band or above) and which is designed to have a large unambiguous range, it is difficult to avoid this blind speed problem, in particular when the radar is required to detect targets with very large velocities. One of the approaches to alleviate this problem is to use a staggered-pulse MTI system in which the inter-pulse durations vary from pulse-to-pulse.

The design of such filters has been discussed in the literature (for example in references [1,2,3]). However, these design approaches are concerned only with the filter weights, with the assumption that the interpulse durations are fixed. Furthermore, most other procedures determine a set of weights which yield a good clutter rejection in the stopband region. The ripples and variations of the filter response in the passband are generally ignored.* For a good filter performance, the smoothness of the response in the passband is just as important as the attenuation in the stopband. That is, target detectability should be uniform in the passband. Thus the existing methods solve only part of the problem. A desirable solution yields a filter with maximum attenuation in the stopband and minimum ripples in the passband region. To achieve this kind of characteristic, more degrees of design freedom are required. In the case of the fixed-PRF MTI system, these are provided by including feedback loops in the delay line network. By properly adjusting the locations of the poles and zeros of the transfer function, it is possible to design a filter having specified characteristics. This aspect has been adequately treated in the literature. However, feedback loops generally increase the filter transient response time which is undesirable for radar MTI applications. For a staggered-pulse MTI transversal filter, an additional degree of design freedom is provided by variation of the interpulse durations. Thus at least in theory, one should be able to use this additional degree of freedom to one's advantage. However, achievement of an optimal solution

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*During the preparation of this manuscript a paper by Jacomini, (O. J. Jacomini, "Weighting Factor and Transmission Time Optimization in Video MTI Radar," IEEE Trans Aerospace and Electronic Systems, Vol AES-8, pp 517-527, July 1972) considered variation of the interpulse periods. However, his criterion of optimality is based on minimizing a linear cost function and differs from the criterion presented in this paper.

to such a problem is extremely difficult. In this paper, we attempt to develop an approach for solving such a problem. In the formulation of the optimization procedure, we propose a condition which follows closely the criterion of a conventional comb filter in which the attenuation in the stopband is maximum while variations in the passband are minimum. A solution which is based on a search technique is presented which at least yields a locally optimal solution for the imposed conditions.

POWER TRANSFER FUNCTION AND AVERAGE GAIN

The impulse response of a staggered MTI filter, shown in Fig. 1, can be represented as:

$$f(t) = \sum_{n=0}^{N} a_n \, \delta(t - T_n) \tag{1}$$

where

$$T_n = \sum_{i=0}^{n-1} \tau_i, \quad T_0 = 0$$

and τ_i represents the interpulse duration. The Fourier transform of the above expression is:

$$F(f) = \sum_{i} a_i \exp(-j2\pi f \, T_i) \tag{2}$$

where f is the doppler frequency of the radar return.* The power spectrum density of this transfer function (henceforth, it will be called the power

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*The summation indices in (2) and subsequent equations vary from 0 to N unless stated otherwise.

transfer function), is

$$|F(f)|^2 = \sum_i \sum_j a_i\, a_j \cos 2\pi f(T_i - T_j). \tag{3}$$

The design goal is then to find a set of weights a_i and a set of interpulse durations τ_i such that the power transfer function has the desired characteristics.

Before investigating the design conditions and procedures, it will be informative to first discuss some properties of the power transfer function.

a. Since we are only interested in the relative amplitude of this function, the weights can be normalized by any constant and the characteristics of the function will not be changed.

b. If the interpulse durations are constrained such that

$$f_r\, \tau_i = \ell_i \tag{4}$$

where ℓ_i are integers for all i then f_r is the first blind doppler frequency. This is equivalent to saying that the frequency response of the filter is a periodic function having a period of f_r, or

$$|F(f)|^2 = |F(f + nf_r)|^2. \tag{5}$$

This becomes evident if one replaces f in (3) by $f + nf_r$. For the case that the interpulse durations are fixed having a value τ, then (3) becomes

$$|F(f)|^2 = \sum_i \sum_j a_i\, a_j \cos 2\pi\, f\, (i-j)\, \tau. \tag{6}$$

Therefore, in this case, the first blind doppler frequency occurs at the PRF which is the reciprocal of the interpulse duration τ.

c. The power transfer function is symmetric within each period, that is

$$|F(f_r - f)|^2 = \sum_i \sum_j a_i a_j \cos 2\pi (f_r - f)(T_i - T_j) = |F(f)|^2. \tag{7}$$

d. The average value of this power transfer function is

$$\overline{|F(f)|^2} = \frac{1}{f_r} \int_0^{f_r} \sum_i \sum_j a_i a_j \cos 2\pi (T_i - T_j)\, df = \sum_i a_i^2. \tag{8}$$

e. If one assumes that white noise having a unit spectral density function

$$n_o = 1 \tag{9}$$

is applied to this filter, the ensemble average output noise power is equal to $\sum_i a_i^2$. Thus if the filter weights are normalized by a factor $\sqrt{\sum_i a_i^2}$ the average output is just unity. Since both the average gain of the power transfer function and the noise power gain are equal to $\sum_i a_i^2$, the average S/N is not degraded. Thus, one may define this as an average gain of the power transfer function.

CLUTTER OUTPUT

Assume that the power spectrum density function of the clutter return is G(f), then the clutter output power is

$$C(T_i, a_i) = \int_{-\infty}^{\infty} \sum_i \sum_j a_i\, a_j\, G(f) \cos 2\pi\, f\, (T_i - T_j)\, df. \tag{10}$$

Changing the order of summations and integration, one finds

$$C(T_i, a_i) = \sum_i \sum_j a_i\, a_j\, \rho_{ij} \tag{11}$$

where

$$\rho_{ij} = \int_{-\infty}^{\infty} G(f) \cos 2\pi\, f\, (T_i - T_j)\, df,$$

which is seen to be the clutter covariance function at time $T_i - T_j$. One of the most desirable optimization conditions is to minimize this clutter power output. However, if one attempts to do so, it will lead to a trivial solution that $a_i = 0$. To avoid this situation, one may require that the average gain of this clutter output be some fraction of the average gain of the power transfer function such that

$$\sum_i \sum_j a_i\, a_j\, \rho_{ij} = \lambda \sum_i a_i^2 \tag{12}$$

where λ represents the clutter suppression factor. In order to find a set of a_i with a given set of T_i such that λ is minimum, the following conditions must be satisfied

$$\sum_j a_j\, \rho_{ij} - \lambda\, a_i = 0 \tag{13}$$

$$i = 0, \ldots, N$$

$$j = 0, \ldots, N.$$

This is a typical eigen-value problem. There are (N + 1) such a_i sets which satisfy the above equation. Each of these sets comprise the eigen-vector associated with an eigen-value λ. Thus the eigen-vector associated with the minimum λ is the desired optimal solution. In references [1] and [3], the average S/C gain of the MTI filter is optimized. This is referred to as the reference gain and is numerically equal to the reciprocal of λ. Alternatively, this procedure is equivalent to maximizing the clutter attenuation of a filter normalized to the average power gain.

As an example, assume that G(f) has a uniform distribution function with a limited bandwidth such that

$$G(f) = \frac{1}{f_u - f_\ell}, \; f_\ell \le f \le f_u \qquad (14)$$

$$= 0, \text{ otherwise}$$

then

$$\rho_{ij} = \frac{1}{f_u - f_\ell} \int_{f_\ell}^{f_u} \cos 2\pi f (T_i - T_j)\, df$$

$$= \frac{\sin 2\pi f_u (T_i - T_j) - \sin 2\pi f_\ell (T_i - T_j)}{2\pi (f_u - f_\ell)(T_i - T_j)}. \qquad (15)$$

If $f_\ell = 0$, this then becomes

$$\rho_{ij} = \frac{\sin 2\pi f_u (T_i - T_j)}{2\pi f_u (T_i - T_j)}. \qquad (16)$$

If the G(f) is Gaussian and has a mean f_o and variance σ_c^2, then

$$\rho_{ij} = \cos 2\pi f_o (T_i - T_j) \exp\left[-2\pi^2 (T_i - T_j)^2 \cdot \sigma_c^2 \right]. \qquad (17)$$

In the case $f_o = 0$, this becomes the familiar case treated by Emerson [1] and Murakami [3].

AVERAGE GAIN IN THE PASSBAND REGION

Next, consider the stopband and the passband regions of the power transfer function. Let the stopband region extend from $f = 0$ to f_u and the passband region extend from $f = f_u$ to $f_r - f_u$. Then

$$\int_0^{f_r} \sum_i \sum_j a_i\, a_j \cos 2\pi f (T_i - T_j)\, df$$

$$= 2 \int_0^{f_u} \sum_i \sum_j a_i\, a_j \cos 2\pi f (T_i - T_j)\, df +$$

$$2 \int_{f_u}^{f_r/2} \sum_i \sum_j a_i\, a_j \cos 2\pi f (T_i - T_j)\, df. \qquad (18)$$

Formulation of (18) uses the fact that the filter power transfer function is symmetric about $f_r/2$.

The last term in the above equation expresses the output of a signal which has a uniform spectrum in the passband. This merely states that the doppler shift of the return target signal is not known a priori. Furthermore, the first term in (18) is equal to $\sum_i a_i^2$ from (8), irrespective of the value of a_i. Under this condition, the average output in the passband region is determined from (18) as

$$\frac{1}{f_r/2 - f_u} \int_{f_u}^{f_r/2} \sum_i \sum_j a_i\, a_j \cos 2\pi f (T_i - T_j)\, df$$

$$= \frac{f_r/2}{f_r/2 - f_u} \cdot \sum_i a_i^2 - \frac{1}{f_r/2 - f_u} \int_0^{f_u} \sum_i \sum_j a_i\, a_j \cos 2\pi f (T_i - T_j)\, df. \qquad (19)$$

By regarding the a_i to be derived for a uniform spectral density function of unit amplitude from 0 to f_u and by noticing the relations of (16) and (13), one finds that

$$\frac{1}{f_r/2 - f_u} \int_{f_u}^{f_r/2} \sum_i \sum_j a_i\, a_j \cos 2\pi f\,(T - T_j)\, df$$

$$= \left[1 + \frac{2f_u\,(1 - \lambda)}{f_r - 2f_u} \right] \sum_i a_i^2 \qquad (20)$$

The left side of this equation represents the average response of this filter in the passband region. In the case $f_u << f_r$, which usually is the case when the clutter spectrum is concentrated in the region close to $f = 0$, the left side of the above equation can be approximated by the average gain $\sum_i a_i^2$. Although (20) was derived for a uniform spectral density over the stopband region, the effect of any other spectral density is to modify λ. For any practical filter, $\lambda << 1$, so that the above relation can be generalized to include all other cases.

The mean-squared deviation from this average value in the passband is then

$$\epsilon_r^2 = \frac{1}{f_r/2 - f_u} \int_{f_u}^{f_r/2} \left[\sum_{i \neq j}\sum a_i\, a_j \cos 2\pi\,(T_i - T_j)\right]^2 df. \qquad (21)$$

In order to form a filter that has a response curve with minimum variations in the passband region, this error function must be a minimum.

In order for this to be achieved by choice of a set of T_i, it is required that:

$$\frac{\delta \epsilon_r^2 (T_i)}{\delta T_i} = 0 \qquad i = 0, \ldots, N. \tag{22}$$

Then, at least in theory, with the condition of (22) and (13), one should be able to solve for a set of T_i and a_i which yields a filter design having the desired characteristics; i.e., minimum ripple in the passband and maximum attenuation in the stopband.

Unfortunately, the condition of (22) is not readily solvable in closed form by known methods. In the following section we propose an approximate means to solve this problem.

A SEARCH SOLUTION

In view of the difficulty in seeking a solution for the above problem, we suggest here an alternative means to seek an approximate solution. This method is based on a search alogrithm.

The procedure of this approach is shown in Fig. 2. The index i represents the i^{th} component of τ_i while index k represents the success of each trial. The a_j's represent the filter weights. Index M represents the number of iterations. First an initial set of interpulse durations is assumed. With the assumed interpulse durations and by use of the relation of (14), a set of optimal weights with an associated eigen-value, λ, are found. If this λ is less than the specified clutter attenuation (S.C.A.), these weights are inserted into (21) to compute the value of the error function. Both the eigen-value and the error function are recorded. The next step is then to either increase or decrease the interpulse durations by an amount $\Delta\tau_i$. This is done one interpulse-duration at a time. Again the eigen-value λ is computed which is then compared with the specified clutter attenuation factor (S.C.A.) and the previous recorded λ. If it is improved, then the error function ϵ is computed and the result is compared with the previous recorded ϵ. If ϵ is improved then the new τ_i value is retained and another of the interpulse spacings will be varied. This procedure is repeated until no further improvement in ϵ is obtained and the function has apparently reached a local minimum. The procedure can be either stopped, or a new set of initial values of τ_i can be assumed and the same procedure repeated again. It is assumed that an adequate number of pulses are available to achieve the required clutter attenuation.

Equation (4), repeated below, sets the boundary conditions on the interpulse duration

$$f_r \tau_i = \ell_i \tag{4}$$

where τ_i is the interpulse duration and ℓ_i is an integer. Hence, the variations of τ_i are limited to discrete values at steps which are the reciprocal of f_r. In practice, in order to obtain sufficient energy on a target, the interpulse time variations are bounded and therefore only a few values of τ_i need be considered in using the search program. Moreover, the minimum interpulse time is dictated in practice by the maximum range of the clutter. The constraint which requires that τ_i

be discrete does not necessarily satisfy the condition of (22). By relying on the search method discussed above, we may find a solution which is not necessarily absolutely optimal; however the solution is always physically realizable. Furthermore, this procedure can be repeated, each time with a set of different initial values, until satisfactory results are achieved. In the following section we present several examples.

NUMERICAL EXAMPLES

All examples are for 7-pulse, staggered-PRF, MTI cancellers. To reduce local minimum problems, the search routine has been repeated about ten times. Each time a different set of initial interpulse periods are used. The best results are then plotted as a frequency response curve shown in these illustrations. The initial τ_i values are generated by a random number generator as discussed in the previous section. The whole search routine takes about two minutes on a CDC 3800 machine.

Figure 3 shows a design with a stopband extending from f = 0 to f = .2 of f_d where f_d is a normalizing factor corresponding to the reciprocal of the smallest interpulse period. This is equivalent to a filter notch of approximately 120 knots for an L-band radar at an unambiguous range of 80 nautical miles. The first normalized blind speed occurs at f = 10. The eigen-value which represents the clutter suppression factor is about 0.00004 and the average rms error in the passband is 0.670768. The interpulse durations shown in that figure are normalized with respect to the smallest τ_i, while the filter weights are normalized to the square root of the average gain. At zero dB level, the gain of the power transfer function is equal to the average gain. In the second example shown in Fig. 4, the stopband is set at a range from $f = 0.5$ to $f = 0.657$. This curve clearly demonstrates the periodic property of this type of filter. In these examples, it is assumed that the input clutter power density function is uniform, that is

$$G(f) = 1 \qquad f_\ell \le f \le f_u$$
$$= 0 \qquad \text{otherwise.}$$

However this same procedure can be applied to a clutter power density function with an arbitrary distribution.

SUMMARY

The first order effects of varying the interpulse periods and weights in an MTI transversal filter are to alter the filter response in the passband and rejection regions respectively. The purpose of this paper is to show how an acceptable filter design which is compatible with a radar waveform requirement may be achieved by systematically varying the interpulse periods using a search algorithm to minimize the response variation in the passband or visibility region of the filter while simultaneously selecting a set of weights which maximize the clutter attenuation.

REFERENCES

[1.] Emerson, R. C., "Some Pulse Doppler and MTI Techniques," Rand Report, R-274, March 1954.

[2.] Ares, M., "Some Anticlutter Waveforms Suitable for Phased Array Radars," R66EMH8, G. E., Technical Information Series, November 1965.

[3.] Murakami, T., and R. S. Johnson, "Clutter Suppression by use of Weighted Pulse Trains," RCA Review, Vol. 32, September 1971.

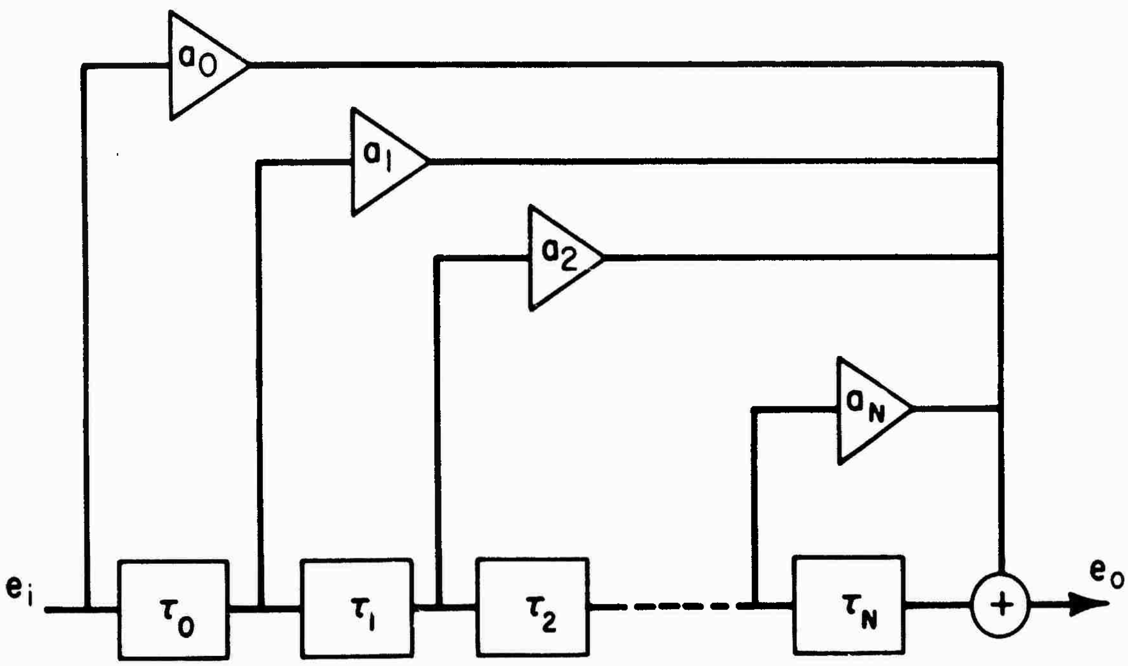

Fig. 1 — A staggered PRF MTI filter

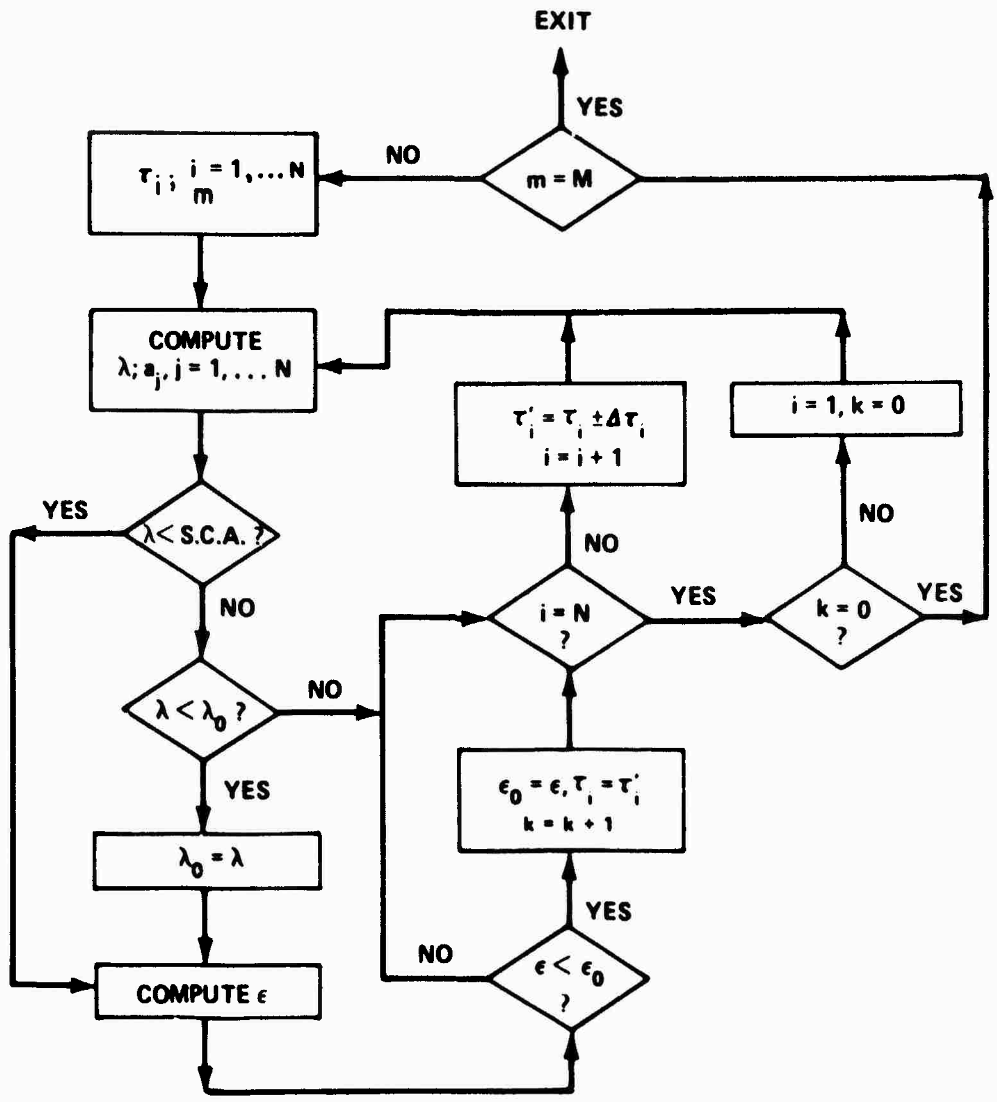

Fig. 2 — Flow chart of search program

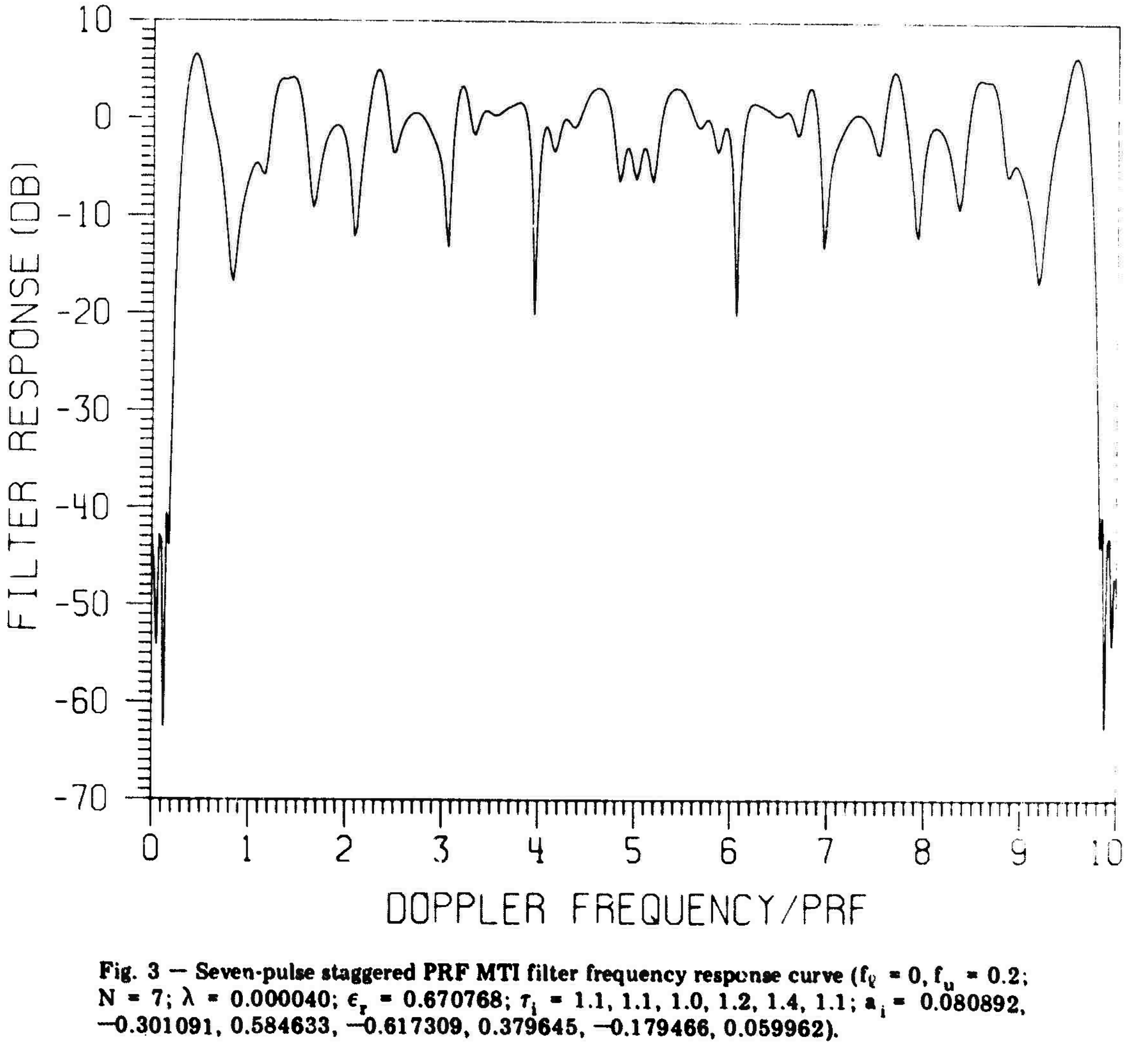

Fig. 3 — Seven-pulse staggered PRF MTI filter frequency response curve ($f_\ell = 0$, $f_u = 0.2$; $N = 7$; $\lambda = 0.000040$; $\epsilon_r = 0.670768$; $\tau_i = 1.1, 1.1, 1.0, 1.2, 1.4, 1.1$; $a_i = 0.080892, -0.301091, 0.584633, -0.617309, 0.379645, -0.179466, 0.059962$).

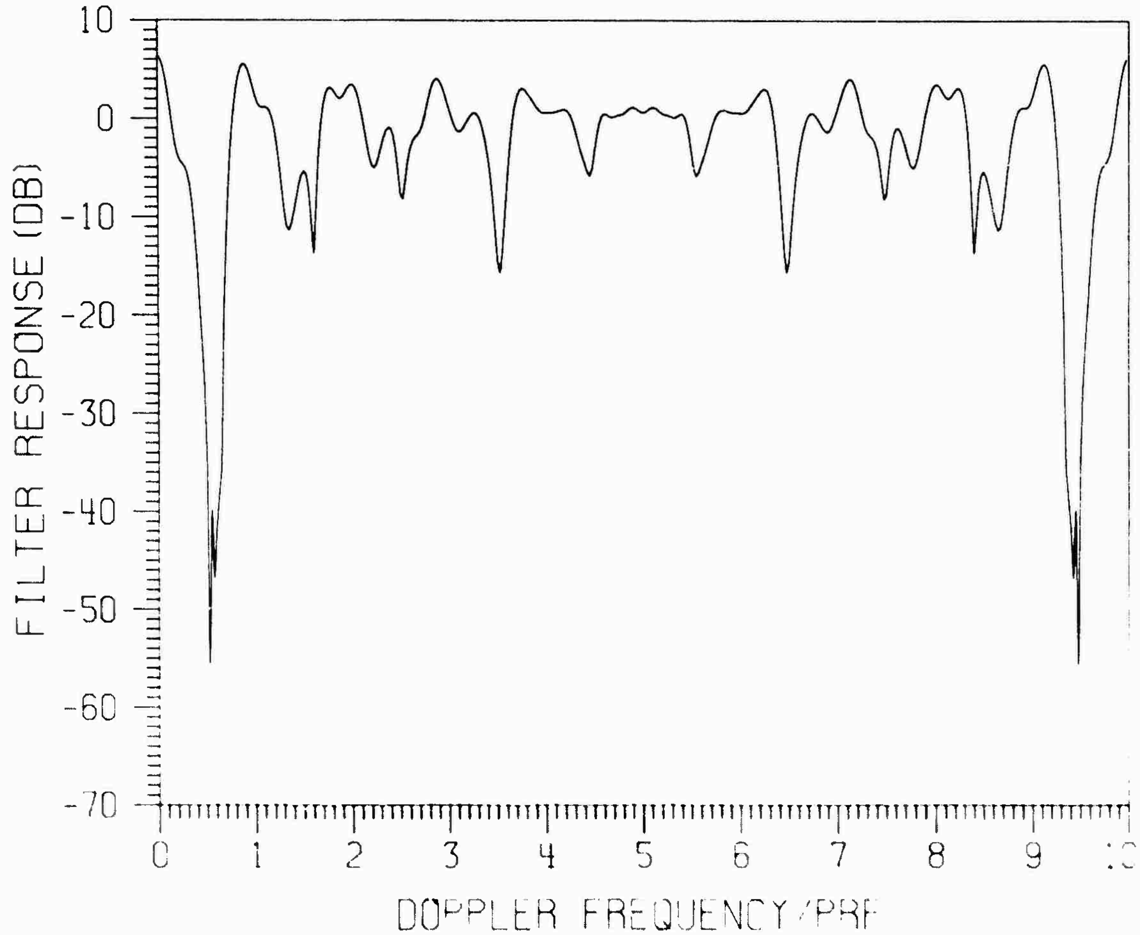

Fig. 4 — Seven-pulse staggered PRF MTI filter frequency response curve (f_c = 0.5, f_u = 0.657; N = 7, λ = 0.00004; ϵ_r = 0.624926; τ_i = 1.3, 1.2, 1.0, 1.1, 1.3, 1.1; a_i = 0.075466, 0.120090, 0.556937, 0.716912, 0.342963, 0.177940, 0.080346).

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13. ABSTRACT

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